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Paper Authors

*** DR. PAWAN KUMAR, DR.SHAKUNTALA MISHRA.**
* Dept of Mathematics, National Rehabilitation University, Lucknow.

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FLOW IN VISCOELASTIC CAPILLARIES ALONG WITH RHEOLOGICAL MODEL

***DR. PAWAN KUMAR, **DR.SHAKUNTALA MISHRA**
*PG Scholar, Dept of Mathematics, National Rehabilitation University, Lucknow.
**Assistant Professor, Dept of Mathematics, National Rehabilitation University, Lucknow.

ABSTRACT :

The behaviour of Newtonian fluid can be understood by LWR law. The viscoelastic fluid include alginate, xanthan, blood which does not show equal Newtonian behaviour. In the current we study Herschel Bulkely (HB) rheological model also we used Navier stokes equation which derive a generic expression of capillary flow of non-Newtonian fluid after that they will convert in the LWR law for Newtonian fluids. Although HB model will not convert the carson's law which gives the shape to whole blood rheology. The HB Model is accurate from for blood rheology. The derived expression is accurately used for the flow of whole blood. The flow of blood shows Newtonian behaviour.

Keywords : Newtonian fluid, Rheological Model, generic expressions, Whole blood.

INTRODUCTION :

The Newtonian fluid in the form of capillary flow are studied in large extent in the current studied in different systems including cylindrical duct along with law of Lucas – Washburn-Rideal [5-8] and second systems is in the form of Channel geometry [9, 10] the current study develop the flow of capillary with non uniform channels [11-14] the flow of blood through capillary get special attention with care and in home care system also because this system show different significant features like minimum cost, it is portable system, it is energetically autonomous and eco-friendly. The whole blood exhibits different systems related with the study of blood having shear rate [1-4]. The nature is depends on the shearing rate. The blood exhibits a Newtonian behaviour & its viscosity μo is given by

$$\mu o = \frac{\tau}{r}$$

Where

γ = Shearing rate

τ = Shearing stress

If there is lower the shearing rate the blood show Non-Newtonian rheology. Also when the shear stress is not much sufficient the fluid acts as a solid with an infinite viscosity. In case of medium flow velocities for diluted blood cells tends to migrate away from the walls forming a cell free layer. But this effects in case of capillary flow show that the velocities are small.

In current work the Herschel Bulkley rheological model [15] is used to derive a general equation for the dynamics of capillary flow for non-Newtonian fluids. Recently studies were conducted on capillary flow for power-law fluid [17-18] although the researchers used blood for some of their experiments and their theoretical model could not take the effect of the yield stress into account the current work is based on HB law and provides a very general model for the flow of non-Newtonian fluids with or without stress.

Objectives :

i) To provide general model for the flow of non-Newtonian fluids.
ii) To derive general equation for dynamics of capillary flow for non-Newtonian fluids.
iii) To distinguish the two characteristics rheological behaviours of the blood.

Application :

i) This study is use full for finding the differential equation for dynamics of non-Newtonian capillary flow.
ii) It is help full for demonstrating the relationship between flow of velocity and its shear rate.

Material and Methods :

Observing the Process of Capillary filling for the experiment of flow of blood a special instrument is necessary which is tubular micro channels made up of glass having internal radius 55 µm. the radius of the instrument depends on micro channels' which was used for in vitro experiment into the point and care of the instrument the micro channel have ability to binding the two solid material together at their surface this instrument is also one outlet channel with having width of 1.5 mm and length is 10.2 mm the reservoir of channel 10.5 mm diameter for their millation they used CNC milling machine. The volume of original reservoirs is big as compared as compared to 1.8 µL during the experiment it is also checked that air liquid interface becomes constant and a Laplace pressure is considered as zero or negligible by using holder which is made up from plastic we can tap the reservoir by attaching itself with mille meter paper for avoiding displacement during experiment. During the process of capillary filling with the help and canon EOS GOOD we are observe the filling distance.

COLLECTION OF BLOOD SAMPLE :

For this experiment we collected the blood sample from diseased free donor which is collected in Ethylene demine tetra acetic acid in the vacationer tubes but it is not directly used for experimentation process. It keeps in a storage condition for 3 day at 5^0 C temp for this experiment we maintain the surface tension of blood for 62 n N/M

Mathematical Modelling and consideration :

The Herschal Bulkley model

In the present work the general HB law is applicable for the model for its un Newtonian behaviour the whole blood is necessary for obtaining the rheological constants including $\tau_{0,}$ K, n [30]

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Blood rheological parameters

τ_0 = 0.01395 Pa

K = 0.0103 $Pa.S^n$

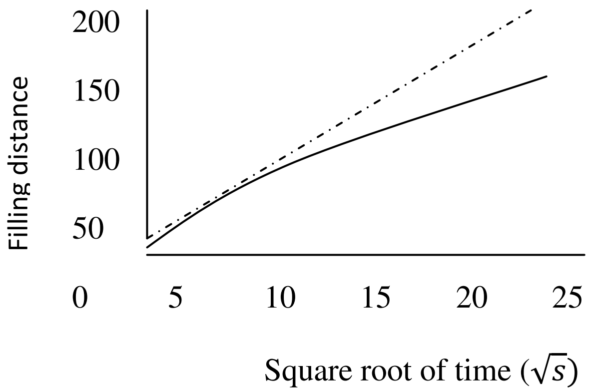
n= 0.782

Non-Newtonian behaviour by using Navier-stokes equation we obtain differential equation for dynamics of the non-Newtonian flow in the capillary although borne rheological parameters are known but it is also true that some coefficients are unknown which include contact angle between blood and Borosilicate tube for calculating the value and the mean square error among experimental result and the solution of model counted with changing between 55^0 C to 75^0 C the values were determined by using software of MATLAB software using ode-45 characters. the contact angle is than the value and smallest mean square error by this value we can plot the graph of the filling distance versus the square root of time we can observe that the initial points are starts to deflects from its linear relationship characterization the Newtonian regimes around by 7.5 cm. from the inlet thus only the data point at a distance farther that 7.5 from inlet are used for non-Newtonian fitting.

Fit of Newtonian flow:

By using Microsoft Excel software the fitting of Newtonian part of the flow is done the deep study of blood rheology at the starting of the flow of capillary when the velocity of the flow is maximum and shear rate dispersed the RBC shows the Newtonian nature of Blood it is also seen that the points ware deviated from Newtonian regime only after 7.5 cm and the transition regimes can be expected during some centimetres. it means that the Newtonian fitting used the points up to 5 to 6 cm. the beginning of the flow of capillary inertia and dynamic of contact angle influence the dynamics [23, 24] the contact angle is equals to static contact angle [20] this to neglect the impact of Newtonian starts only after first two to three centimetres.

Result and Discussion :



The Newtonian -Non-Newtonian transition :

By observing the graph we can find out the two different behaviour this transition of behaviour is consistent with expected with behaviours of undiluted whole blood. indeed at the initial state and the capillary flow there was small friction with the walls and velocity and blood flow which seems to be nigh which forms minimum shear rate which spread the Red blood cells and the blood show Newtonian behaviour [20,27,28] It is also seen that the in the capillary channel by increasing the dray velocity the shear rate also decreases by showing red blood cells aggregating (20-12) [28-14] which show Non-Newtonian behaviour of blood flow In the first part gives the Newtonian behaviour of blood flow which explain that the shearing rate is linear with radius of capillary channel and is obtained through Newtonian parabolic Poiseuille profile and if there was much decreased in shearing rate which permit the formation of rouleaux, can deduced the Newtonian and non-Newtonian transition

In the normal uniform channel the equation of the capillary flow and a Herschel Bulkley fluid is stated by following differential which obtained through Navies-Stokes equation.

$$\frac{d^2x}{dt^2} + \frac{Pw}{\rho A}\frac{K}{(\lambda n)}\frac{(dx)^{\eta}}{(dt)} - \frac{\sigma Pw\ cose*}{\rho Ax(t)} = \frac{Pw}{\rho A}\tau o \text{--------------}(1)$$

Where X = Filling distance

t = Required time

Pw = Wetting Perimeter

A = Cross Section Area

λn= Average Friction Length (Appendix A)

The extension to the friction length was introduced in [8]

σ = Surface tension

θ = Cassie angle

ρ = Fluid density

There are also some various nominations which was stated in Appendix A The Problems having complexity is hidden in the mean generalised friction length λn the mathematical expression for it is deduced in App B section which was in the case of cylindrical duct.

Equation (1) becomes

$$\frac{d^2x}{dt^2} + \frac{2}{\rho R}K\ (\frac{n+1}{(h}\ \frac{1^{\eta}}{R\alpha)}\ (1-c) - (\frac{dx}{dt}) - \frac{2r\ cos}{\rho Rx(t)} = \frac{2Pw}{\rho R}\text{--------------}(2)$$

R = Radius of channel

α and c = dimensionless coefficient (app B)

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In the above expression the internal terms are avoid because in the micro channels the Raynolds and Weber numbers are less and they show small shearing value

so the above equation becomes

$x\left(\frac{dx}{dt}\right) = \frac{1}{1-c}\frac{1}{K(\frac{n+1}{R})}(\Upsilon\cos\theta - x\tau_o)$ -------------------------(3)

For Newtonian Fluid

n=1, τ_0=0, α=1/2, K=μ

The above eq^n is not applicable for Herschel - Bulkley fluid

The newly obtained eq^n is

$x^{\frac{1}{n}}\frac{dx}{dt} = [\frac{1}{K\left(\frac{n+1}{nR\alpha}\right)}\Upsilon\cos\theta]^{\frac{1}{n}}$------------------(4)

By integrating we get closed form

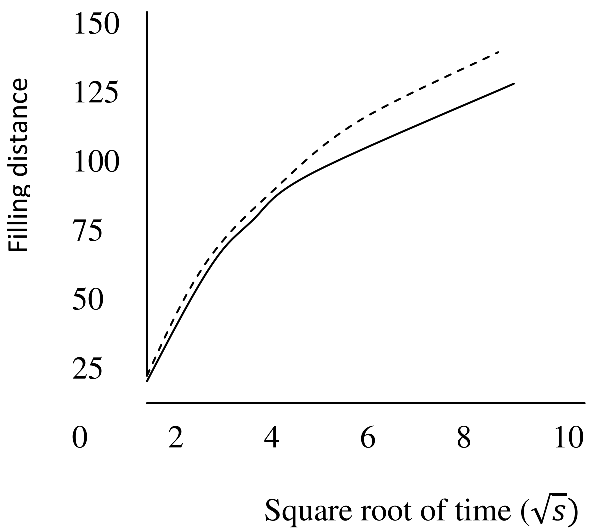
X= $\left[\frac{n+1}{3n+1}\frac{Rn}{K}\Upsilon\cos\theta\right]\frac{1}{n+1}t\frac{n}{n+1}$ ------------------(5)

Non - Newtonian regime :

The filling of non-Newtonian part and flow is obtained through angle of contact and mean square error (MSE) which was given following

	Channel 1	**Channel 2**	**Channel 3**
Contact angle	54.8^0	55.9^0	55.1^0
Mean Squar error x $(10)^{-5}$	2.9	0.5^{30}	2.4

Channel 1



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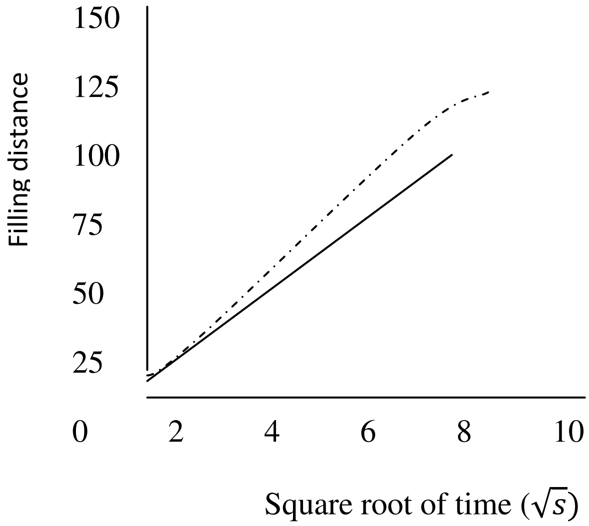
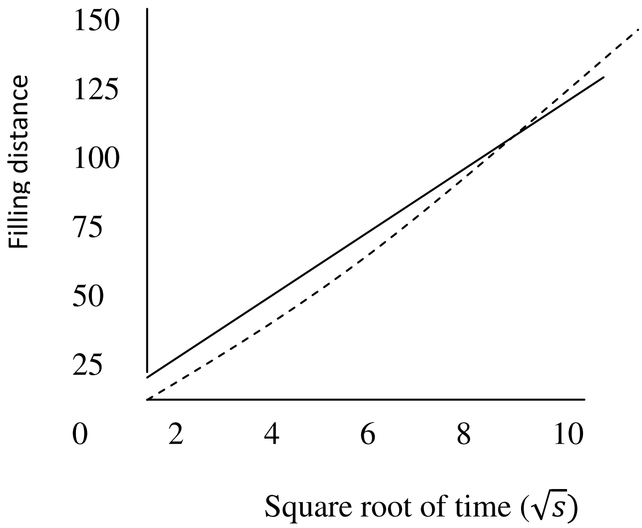
Channel 2



Channel 3



The dashed line show Newtonian fit and the dark line show non-Newtonian model of capillary flow

The Newtonian regime

The law for the filling of cylindrical duct through Newtonian fluid is given by [11-15]

$$x=\sqrt{\frac{v}{\mu}\,\frac{R}{z}\,cos\theta}\;\sqrt{t}\text{------------------ (6)}$$

Where the viscosity is given by

$$\mu=\frac{R*V*cos\theta}{2*\beta^2}$$

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The value and viscosity obtained from lihear fit and Newtonian part is given below

	Channel 1	Channel 2	Channel 3
Viscosity (mPa.s)	3.57	3.65	3.78

From above table we get Newtonian fits in three forms with ref. [22] the value obtained in current study gives the blood viscosity value by showing maximum shearing rate.

CONCLUSION :

In the current study we get the complicated composition of blood which gives two systems based on shearing rate first is Newtonian system and second is non-Newtonian system which show small shear rate In the current study we investigated the filling of capillary with a whole blood we also study some laws which explain the linearity and the square root of time we get the Herschel-Bulkley fluid eq^n of capillary through Navier stokes equation this view demonstrate the time period of equation of non-Newtonian of capillary flow

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Appendix

Appendix A

Dynamics of the capillary flow of Herschel Bulkley fluid

We know that the general Navier-Stokes eq^n is

$$\rho\frac{D\nabla}{Dt} = -\vec{\nabla}p + \vec{\nabla}.\tau + \rho f \text{-----(1)}$$

Where

ρ = fluid density,

v = flow velocity

t = time period,

p = Pressure,

τ = stress tensor,

f = force applied by body

If no applied external force is applied & $\frac{dv}{dx} = 0$

we get $\rho\frac{dv}{dt}= -\frac{dp}{dx}+ \nabla \tau_x$ ----------------- (2)

by applying Gauss Ostrogradisk th^{m} & integrating we can find wall friction i.e.

$= \frac{1}{Pw}\, \varphi_t\, \tau w dl$ ------- (3)

Where $\tau w = \left(\frac{Co}{1 v\omega 1} + K|\,\Upsilon w|^{n-1}\right)\Upsilon\omega$

& $v = -\lambda\frac{dv(r)}{d\eta}\,\omega$

the average fiction length λn is given by

$$\left[\frac{1}{\lambda_n} = \frac{1}{Pw}\,\varphi_\tau\,\frac{1}{\lambda_n}\,dt\right]$$

Appendix B

Determination of average friction length the average velocity equation is-

$v=\frac{v}{\alpha}\left[(1-c)^{\frac{n+1}{n}} - (\frac{v}{R} - c)^{\frac{n+1}{n}}\right]$ for r > Ro ∴

& $v = \frac{v}{\alpha}(1-c)^{\frac{n+1}{m}}\left\{1 - 2(1-c)(\frac{n}{3n+1}\,(1-c) + c\,\frac{n}{2n+1})\right\}$

$C=\frac{R0}{R}$ & $Ro = -\frac{2\tau o}{G}$

now the fringe length is

$$\frac{1}{\lambda n} = \left(\frac{n+1}{n} - \frac{1}{R\alpha}\right) \quad (1-c)^{\frac{1}{n}}$$

average friction length is

$$\frac{1}{\lambda n} = \frac{1}{Pn}\,\varphi_\Gamma\,\frac{1}{\lambda n}\left(\frac{n+1}{n} - \frac{1}{R\alpha}\right)^{n}(1-c)$$