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Paper Authors
Chivukula V S D Kartheek, P. Vishwanath Kumar

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Modelling and Simulation of an AutonomousQuadrotor UAV

Chivukula V S D Kartheek[1], P. Vishwanath Kumar[2*]
[1]Production Planner, TechnipFMC, Hyderabad, India
[2]Department of Mechanical Engg., Anurag University, Hyderabad, Telangana 500088, India
*Corresponding author. E-msail: vishwanathmech@anurag.edu.in
Contributing author: kartheekc19@gmail.com

Abstract
The aim of this paper is to design and study two different control strategies for a quadcopter system. For this purpose, a six degrees of freedom non-linear dynamic model of a quadcopter model is developed using Newton-Euler approach. The PID controller and the Linear Quadratic Regulator (LQR) techniques are used to control the quadcopter to reach desired attitudes. A 3-dimensional curve equation is used as a path for the quadcopter and a PID controller is implemented to fly along the specified path. The PID controller is implemented on the non-linear system by decoupling the lateral and longitudinal dynamics and by using a distinct controller for individual attitude variables. A linearized model of the quadcopter is obtained and the Linear Quadratic Regulator (LQR) controller is implemented. Firstly, the controller is tested under the situation where the wind perturbations are absent. Then, process noise is added along with the control input and the desired LQR controller is tested under this situation. A comparative study is also done on the above-mentioned scenarios. The suggested control algorithms are evaluated on a quadcopter model using numerical simulations in MATLAB/Simulink, with various simulation settings being examined to show the validity and efficiency of the proposed controllers.

Objectives

1. To develop a six degrees of freedom non-linear dynamic model of a quadcopter with

   0.2 kg weight and 0.225 meters arm length.
2. To design the controllers to follow a specified path and study its state variables overtime.
3. To conduct a performance study on two different linear controllers on the six

degreesof freedom non-linear dynamic model of a quadcopter.

4. To perform a comparative study on the LQR controller during the absence and presenceof wind disturbances.

Introduction

An Unmanned aerial vehicle (UAV) is an aircraft that can be controlled wirelessly or can traverse autonomously. The applications of these UAV systems are growing exponentially in many areas such as surveillance, agriculture and disaster control etc. A quadcopter is a UAV that has four propellers with the propeller speeds as control inputs. Due to their small size, quadcopters provide high mobility and can traverse through complex trajectories.

Before designing the controller, the dynamics of the quadcopter system must be studied. The dynamic equations of the quadcopter are derived using Newton-Euler and Euler-Lagrange equations for 3- dimensional motion of a rigid body. Using small disturbance theory, a linearized model has to be derived from the non-linear model by linearizing the equations around a hovering point (Hajiyev, C. and Vural, S,2013). The characteristic values, such as the aerodynamic coefficients, of the quadcopter had to be investigated to study the quadcopter's behaviour. Finally, the controller can be designed based on the required specifications.

Attitude stabilisation and trajectory tracking are the two primary difficulties in quadcopter control (Suresh, H *et al.*,2018). For quadcopters, a number of control techniques for stabilisation and trajectory monitoring have been proposed. The objective is to devise a control system that will allow the states of a quadrotor to converge to any set of time-varying reference states. Many previous research (Jategaonkar, R. V,2004 ; Cowling,I.D *et al.*,2007, Bouabdallah, S *et al.*,2004 ; Pounds,P *et al.*,2006) have demonstrated that by linearizing the dynamics around a hovering point, linear control methods may be employed to regulate the quadcopter. Nonlinear control approaches, on the other hand, can give a broader flying envelope and higher performance by addressing a more generic form of the quadcopter's dynamics under all operating scenarios. Quadrotors have shown to benefit from backstepping (Tarek, M. and Abdelaziz,B,2006 ; Bouabdallah, S. and Siegwart, R,2005), sliding mode (Xu, R. and Ozguner,2006 ; Lee, D *et al.*,2009), and feedback linearization (Das, A *et al.*,2009) among these nonlinear approaches.

In this work, two linear controllers have been implemented on the non-linear quadcopter system: PID controller and Linear Quadratic Regulator (LQR) controller. PID controllers are frequently employed in various industrial applications (Astrom, K. J. and Hagglund, T,1984 ; Ho, W. K *et al.*,1996). The main reason for this is that it has a very straightforward approach that is simple to comprehend and execute in practise (Wang, Q. G *et al.*,1999). The extensive use of PID controllers in industry has influenced attempts to build and tune traditional PID controllers to achieve optimal control system performance (Cheng, Y. C. and Hwang, C,2006). However, due to the linearized approach, PID cannot achieve robust performance, regardless of whether the controlling parameter is Euler angle or angular rates (Wang, P *et al.*,2016).

One of the most widely utilised optimum control techniques for linear systems is the Linear Quadratic Regulator (LQR). To determine the best control decisions, a cost function is considered. The cost function is influenced by the dynamical system's states as well as the control inputs of the quadcopter. LQR can be implemented directly on the non-linear model on contrast to the PID controller. PID controllers have the major drawback of requiring linearization for each test on the actual system. For LQR control, this step is not necessary, and the system equations canbe implemented directly into the controller to achieve the desired response.

The following is a breakdown of the paper's structure: The quadcopter is mathematically modelled in Section 2 using Newton-Euler equations. The PID and LQR controllers are presented in Section 3. The quadcopter's MATLAB / Simulink simulation results are discussed in Section 4, followed by a comparative study. The paper's conclusion and future scope are found in Section 5.

Mathematical Modelling

Description

The quadcopter consists of four independently controllable rotors. The quadcopter's movement is caused by changes in the rotors' speeds. In this study, the quadcopter structure is considered to be rigid and symmetrical, with the centre of gravity and the body fixed frame origin aligned, the propellers are also considered

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to be rigid, and thrust and drag forces proportional to the square of the propeller's speed (Soufiene, B. and Rabii, F,2018). The illustration of a quadcopter and relative coordinate systems are shown in Fig 1 below.

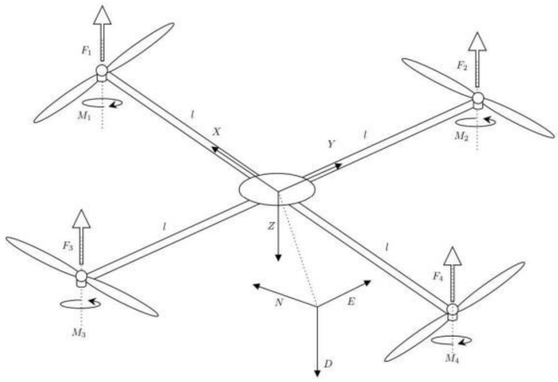

Fig. 1 Quadcopter system illustration with its relative coordinate systems

Kinematics of the Quadcopter

Let $[x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T$ be the linear and angular velocities of the quadcopter in the earth frame/inertial frame and $[u \quad v \quad w \quad p \quad q \quad r]^T$ be the linear and angular velocities in the body frame.

The matrices $\mathbf{v} = [x\ y\ z]^T \in \mathbb{R}^3$ and $\boldsymbol{\omega} = [\phi \quad \theta \quad \psi]^T \in \mathbb{R}^3$ respectively are used to describe the linear and angular velocities of the quadcopter in the inertialframe.

The two reference frames are linked by using following relations:

$$\mathbf{v} = \mathbf{R} \cdot \mathbf{v}_B$$
$$\boldsymbol{\omega} = \mathbf{T} \cdot \boldsymbol{\omega}_B \qquad (1\ \&\ 2)$$

where $\mathbf{v}_B = [u \quad v \quad w]^T \in \mathbb{R}^3$ and $\boldsymbol{\omega}_B = [p \quad q \quad r]^T \in \mathbb{R}^3$ are used to describe the linear and angular velocities in the body frame respectively.

where R and T are respectively the rotation and transformation matrices between body and inertial frames.

$$\mathbf{R} = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \quad (3)$$

$$\mathbf{T} = \begin{bmatrix} 1 & s(\phi)t(\theta) & c(\phi)t(\theta) \\ 0 & c(\phi) & -s(\phi) \\ 0 & \frac{s(\phi)}{c(\theta)} & \frac{c(\phi)}{c(\theta)} \end{bmatrix} \quad (4)$$

$C \cdot= \cos(\cdot)$, $S \cdot= \sin(\cdot)$ and $T \cdot= \tan(\cdot)$

Dynamics of the Quadcopter

The dynamic model of the quadcopter considering the external forces and moments along with the actuator dynamics can be given by (Priyambodo, T.K *et al*.,2020):

$$\ddot{x} = (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi)\frac{F}{m} \qquad (5)$$

$$\ddot{y} = (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)\frac{F}{m} \qquad (6)$$

$$\ddot{z} = -g + (\cos\phi \cos\theta)\frac{F}{m} \qquad (7)$$

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}} qr - \frac{Jr}{I_{xx}} q\omega + \frac{u_2}{I_{xx}} \qquad (8)$$

$$\dot{q} = \frac{I_{zz}-I_{xx}}{I_{yy}} pr - \frac{Jr}{I_{yy}} p\omega + \frac{u_3}{I_{yy}} \tag{9}$$

$$\dot{r} = \frac{I_{xx}-I_{yy}}{I_{zz}} pq + \frac{u_4}{I_{zz}} \tag{10}$$

where Jr denotes rotor inertia and p, q, and r denote angular velocities in roll, pitch,and yaw motions, respectively.

The actuator dynamics are described as follows (Priyambodo, T.K *et al*.,2020):

$$u_1 = K_f * (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \tag{11}$$

$$u_2 = K_f * (\omega_4^2 - \omega_2^2) \tag{12}$$

$$u_3 = K_f * (\omega_1^2 - \omega_3^2) \tag{13}$$

$$u_4 = K_m * (\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \tag{14}$$

where Kf denotes the thrust factor and Km represents the drag factor, whose value is dependent on the propeller size and the environment conditions in which the quadrotor is operating, and ω_i denotes the ith rotor's angular speed. (Priyambodo, T.K *et al*.,2020).

State-space modelling

There are 12 state variables, out of which four are input variables, and four are output variables in the quadrotor state- space system. The state variables include linear and rotational coordinates, as well as their related velocities, that indicate the absolute quadrotor orientation in space. The input variables are the four quadrotor motions: thrust, roll, pitch, and yaw. The height (z), roll (ϕ), pitch (θ), and yaw (ψ) angle displacements are the required state variables for the quadrotor's stability analysis in the output. U is the matrix having all the control inputs, and X is the state matrix having all the state variables ().

$$X = AX + BU \tag{15}$$

$$Y = CX + DU \tag{16}$$

$$X^T = [x \quad y \quad z \quad \phi \quad \theta \quad \psi \quad x \quad y \quad z \quad p \quad q \quad r] \tag{17}$$

$$U^T = [u_1 \quad u_2 \quad u_3 \quad u_4] \tag{18}$$

$$Y^T = [Z \quad \phi \quad \theta \quad \psi] \tag{19}$$

where **A** is a 12×12 state matrix, while **B** is a 12×4 input matrix which are defined as (Saraf, P *et al*.,2020):

$$\mathbf{A} = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&\frac{u_1}{m}&0&0&0&0 \\ 0&0&0&0&0&0&-\frac{u_1}{m}&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix} \tag{20}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_{xx}} & 0 & 0 \\ 0 & 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix} \quad (21)$$

As previously stated, the C matrix is a 4×12 matrix with a unity multiplier in each column to scrape out the four state parameters from X. D is a null matrix of 4×4 elements. The controller for the quadcopter system is designed once the state-space equations have been found. The next section discusses the overview and design approaches of the control strategies (Saraf, P *et al*.,2020).

Controller Design

Proportional Integral Derivative (PID)control

One of the most extensively used control models is the PID controller. By modifying the proportional, integral, and derivative coefficients in the controller equation, the objective of the controller is to decrease the error to the least possible value. The error is defined as the difference between the setpoint value and the actual controller output at any given moment. The purpose of the proportional term is to raise or lower the error by multiplying the error with a proportionality constant. The derivative term estimates the future response of the controller based on the rate of change in error. A weighing term, known as integral coefficient is used to weight the integral term and is used to obtain the desired output response in the shortest time. All of these coefficients must be tweaked to obtain the setpoint of the control system (Saraf, P *et al*.,2020).

The controller response is computed using the Kp, Ki, and, Kd parameters and the error value in the PID control equation that is given below (Saraf, P *et al*.,2020):

$$U \mid (t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \quad (22)$$

The Kd and Ki terms are given as (Saraf, P. *et al.,* 2020):

$$K_d = K_p T_d \& K_i = K_p / T_i \quad (23)$$

Td and Ti are considered as the time periods for integral and derivative responses. The error value is defined as (Saraf, P. *et al.,* 2020):

(t) =setpoint value - actual value (24)

PID controller design

The PID controller is implemented on the non-linear quadcopter system by decoupling the lateral and longitudinal dynamics and applying a separate controller for each attitude variable. Depending on the action, the four inputs thrust, roll, pitch, and yaw motion govern the six-state parameters (Saraf, P. *et al.,* 2020). Thrust command controls the height of the quadcopter which is the z parameter, roll controls the x and the θ parameters, pitch controls y and ϕ parameter and yaw controls the ψ angle. The attitude controller is a PD controller summed up with the mass feedforward. Rest of the attitude controllers are cascaded PD loops, where the outer PD control loop generates the input reference signal for the inner angle PD control loop. The thrust controller is illustrated in the Fig 2 and the pitch controller is illustrated in the Fig 3.

Linear Quadratic Regulator (LQR) control

In comparison to a normal PID controller, the LQR controller demands a large number of mathematical calculations to create the entire state feedback matrix K. The LQR controller uses an optimal control method to minimise the cost function generated by the system equations. The cost function includes the system's state and input parameters, as well as the Q and R matrices. The entire cost function must be as low as possible for the optimum LQR solution. The Q and R matrices represent the weights applied to the state parameters and input parameters. By altering the values of the two matrices, the total value of the cost function may be changed to get the desired result (Saraf, P. *et al.,* 2020).The control architecture for the LQR controller is illustrated in the Fig 4.

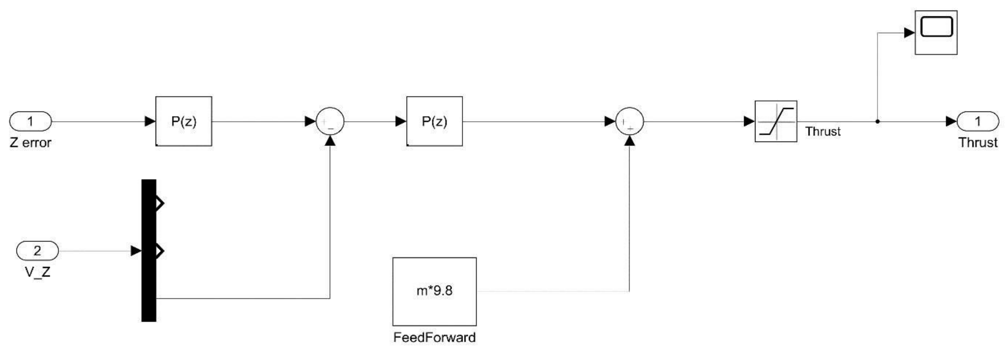

Fig. 2 Thrust controller block diagram

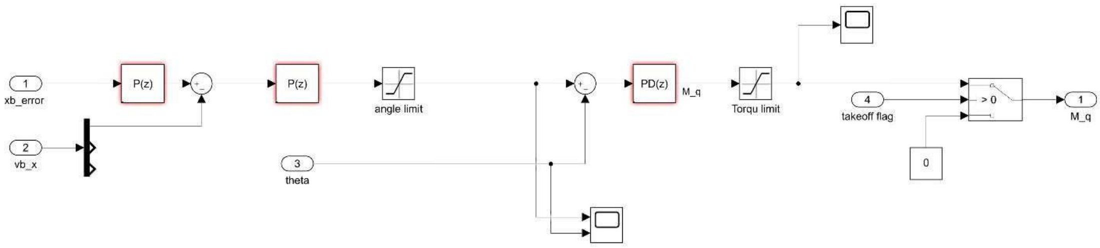

Fig. 3 Pitch controller block diagram

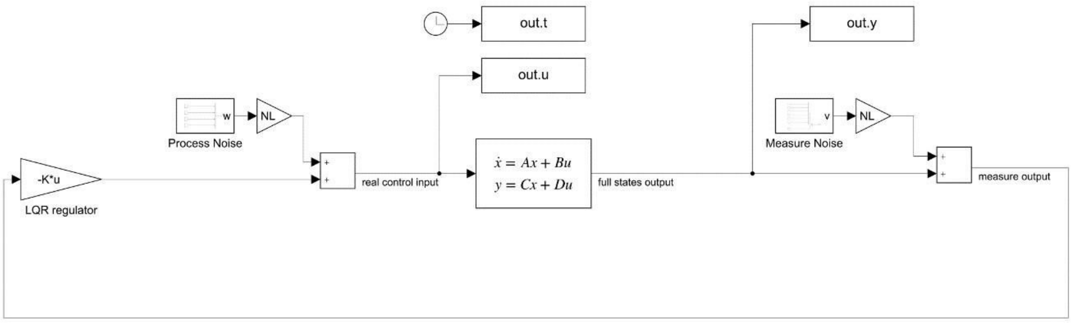

Fig. 4 LQR controller quadcopter system architecture

The LQR is an optimization problemthat requires a linearized state-space model of the system which has been derived in Section 2.4. The cost function of the LQR model is given by:

$$J = \int (X^TQX + U^TRU)t \quad (25)$$

The matrices are need to be tuned according to the desired output matrix. Then, to compute the gain matrix, the Q and R matrices are used to solve the Algebraic Riccati Equation (ARE). The gain matrix is essentially the LQR controller.

$$A^TS + SA - SBR^{-1}B^TS + Q = 0 \quad (26)$$

The S matrix which is obtained from the ARE equation is employed to calculate the full state feedback gain matrix K using the equation:

$$K = R^{-1}B^TS \quad (27)$$

The final feedback control equation is:

$$U = -K * X \quad (28)$$

The next section consists of the simulation results of both the controllers under different conditions.

Results and Discussion

Using the dynamic equations of the quadcopter which are derived in the Section 2, a 6 degrees-of-freedom non-linear quadcopter model is developed in

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the MATLAB / Simulink. The physical parameters of the quadcopter taken into account for the simulation is given below inthe Table 1.

Table 1 physical parameters of the quadcopter

Variable	Value	Units
g	9.8	m/s^2
m	0.200	kg
L	0.225	M
I_{xx}	0.1	kg/s^2
I_{yy}	0.1	kg/s^2
I_{zz}	0.15	kg/s^2

A 3-Dimensional curve equation has been specified as the path for the quadcopter. The initial conditions of the quadcopter are set to zero for all state parameters. The curve equation is transformed to various set points for the quadcopter to achieve and those way points are fed to the quadcopter model as an input. The PID controller takes the error commands of the state variables and generates thrust and attitude moments. These values are now fed into the quadcopter dynamic model and the quadcopter reaches the next state. The simulation time is t =32s and Fig 5 shows the path generated by the quadcopter is illustrated in a 3D plot using MATLAB.

PID controller results

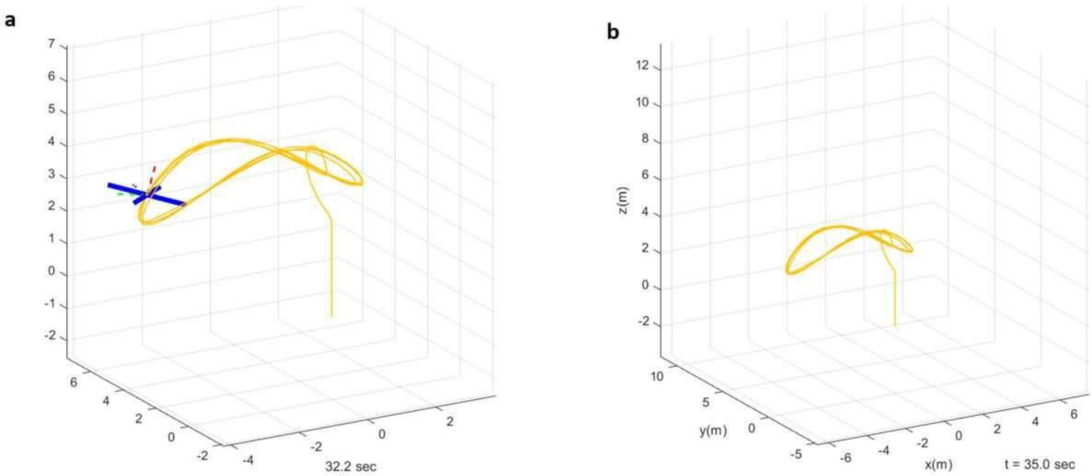

Fig. 5 (a) quadcopter following the trajectory during the simulation. (b) the path generated bythe quadcopter after the simulation

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The desired performance for the quadcopter has been achieved using PID tuning. However, there are slight fluctuations in the y position and high fluctuations in y velocity over time. This indicates a slight underwhelming performance of the PID controller in maintaining its specified path over time. The change in X, Y, Z positions of the quadcopter over time are illustrated in the Fig 7 below and the change in X, Y, Z velocity is shown in the Fig. 8.

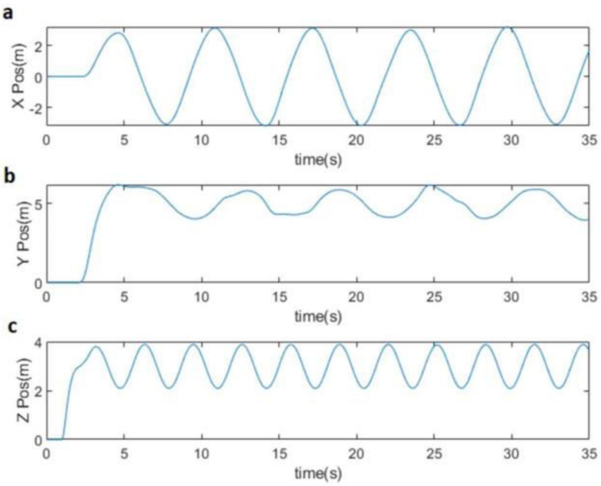

Fig. 6 Quadcopter change in position over time for PID controller. (a) X position over time. (b) Y position over time. (c) Z position over time

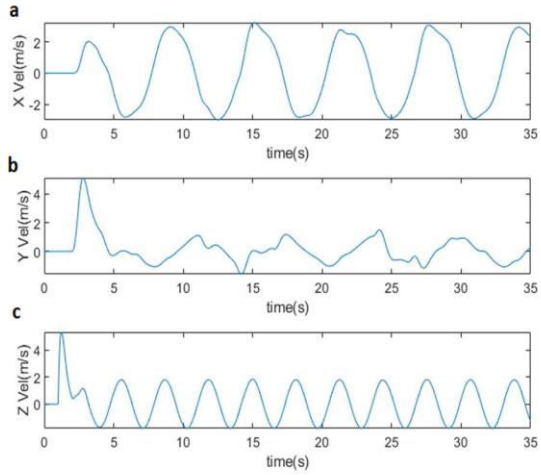

Fig. 7 Quadcopter change in velocity over time. (a) X velocity over time. (b) Y velocity over time. (c) Z velocity over time

Linear Quadratic Regulator (LQR) controller results

The initial conditions for the LQR system have been set to zero for all the state variables. The LQR parameters were derived from the linearized model of thequadcopter as discussed in section 3.2 and this controller is used to control the non- linear quadcopter model. Here, the system is first tested without considering any external wind disturbances.

A square shaped path is specified to the quadcopter system. The path is transformed into set points that are used as an input to the quadrotor model. The simulation time is t = 30s and Fig 8 shows the path generated by the quadcopter is illustrated in a 3D plot using MATLAB.

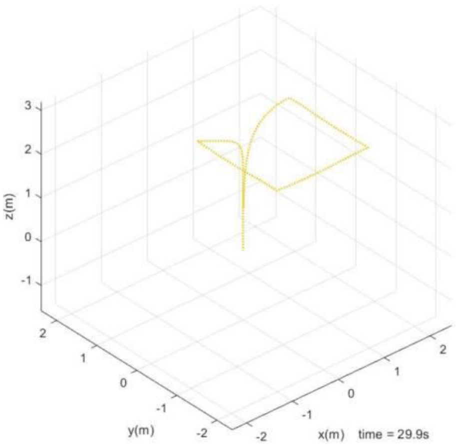

Fig. 8 The path generated by the quadcopter after the simulation

The change in X, Y, Z positions of the quadcopter over time are illustrated in the Fig 9 below and the change in angular rates of the quadcopter is shown in the Fig 10.

After testing the model without any wind disturbances, some process noise is added into the system along with control input. The path specified to the quadcopter is same as the above case. The simulation time is t = 29.9s which is same as the above simulation. The LQR controller is optimized for high performance. Therefore, the change in X, Y and Z positions over the time are exactly same as the above case without any fluctuations. The PID controller fails by having fluctuations for Y position over time without considering wind disturbances, while the LQR controller maintains the same path specified even the wind disturbances are considered.

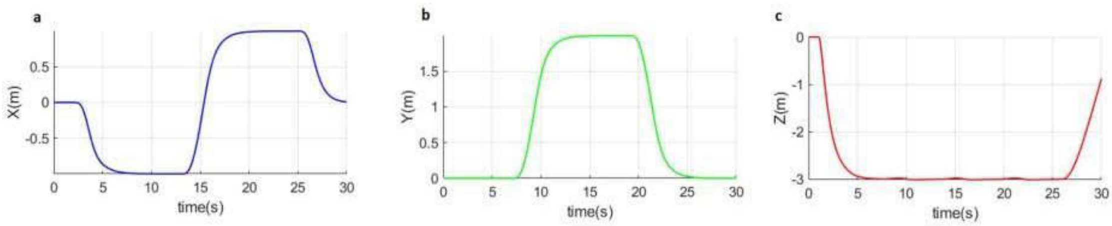

Fig. 9 Quadcopter change in position over time for LQR controller. (a) X position over time. (b) Y position over time. (c) Z position over time

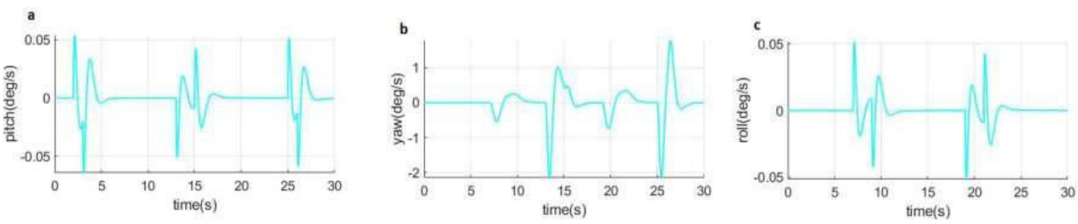

Fig. 10 Quadcopter angular rates over time for LQR controller. (a) pitch over time. (b) yawover time. (c) roll over time

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However, the fluctuations in yaw, pitch and roll angles are very high for the LQR controller with noise when compared to the LQR controller with no noise. Due to the high-performance cost, the model expenses for utilizing high amounts of energy to maintain the quadcopter in the specified path. The quadcopter succeeded to main the specified path amidst these wind disturbances due to this high-performance cost. The squared sum of all the control inputs is considered as a brief criterion for energy. The LQR controller utilizes 56 units of energy when there are disturbances and only 32 units of energy when there are no disturbances. The comparison between both these cases are shown in Fig 11 andFig 12.

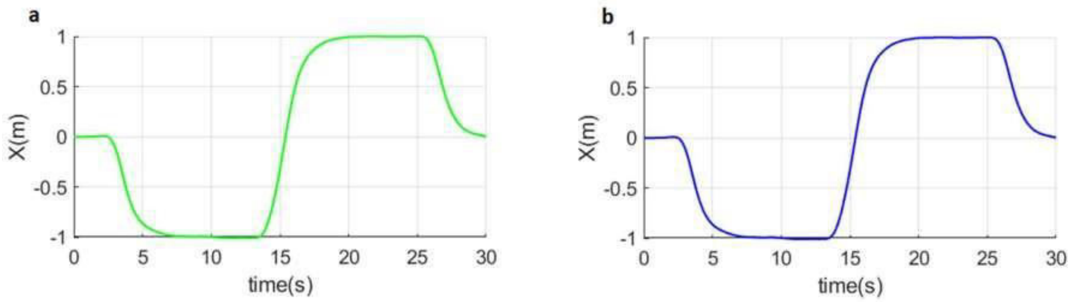

Fig. 11 X position of quadcopter over time. (a) with wind disturbances. (b) without wind disturbances

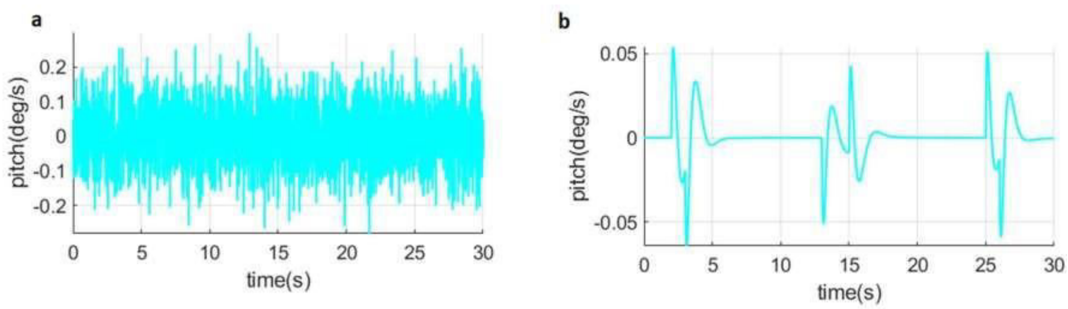

Fig. 12 Pitch angle of quadcopter over time. (a) with wind disturbances. (b) without wind disturbances

Conclusion

This paper presents a performance study on two different linear controllers: PID controller and LQR controller. The six degrees of freedom non-linear dynamic model of the quadcopter is derived using the Newton-Euler equations and is used to test the performance of the controllers. A cascaded PID controller and LQR controller are designed for the quadcopter to follow a specified path. The PID controller slightly underperformed while stabilizing its Y position over time.

To begin, the LQR controller is put to the test the quadcopter in the absence of wind disturbances. After that, process noise is added to the control input, and the system is evaluated with the suggested controller under these disturbances. A comparative study is also done on the above-mentioned scenarios. Due to high performance cost, the quadcopter perfectly followed the path under external wind disturbances and resulted in high fluctuations in the angular rates and also consumed high energy compared to the no wind disturbance case.

The focus of future study will be on incorporating nonlinear control methods with a nonlinear dynamic model in order to increase the quadcopter system's robustness and performance in the face of parameter uncertainty and external perturbations.

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