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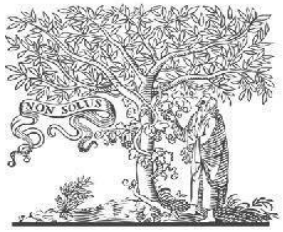

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Paper Authors
S. Hariprasad, D. Giri, Kishore Kumar J, M. Bhupathi Naidu, K. Murali

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Handling Heteroscedasticity: Testing Linear Restrictions in Regression Models

S. Hariprasad[1], D. Giri[2], Kishore Kumar J[3], M. Bhupathi Naidu[4], K. Murali[5]**
[1]Lecturer in Statistics, P.V.K.N Govt. College (A), Chittoor.
[2]Principal, D.K. Govt. College for Women(A), Nellore.
[3]Associate Professor of Statistics, Maharani Science College for Women, Bangalore
[4]Professor, Department of Statistics, S V University, Tirupati.
[5]Department of Statistics, S V University, Tirupati.
[**]Corresponding Author: **dr.murali4stat@gmail.com**

Abstract
Heteroscedasticity poses a significant challenge in regression analysis, affecting the efficiency and validity of parameter estimation and hypothesis testing. This study explores methods for testing linear restrictions on parameters in regression models under heteroscedasticity. We examine the impact of heteroscedastic errors on conventional tests, such as the Wald test, Lagrange Multiplier test, and Likelihood Ratio test, and discuss heteroscedasticity-robust alternatives. Simulation studies and empirical analysis demonstrate the performance of these methods in maintaining statistical validity. The findings provide insights into choosing appropriate testing strategies when dealing with non-constant variance in regression models, ensuring more reliable inference in econometrics and applied statistical research.

Keywords: Wald test, Lagrange Multiplier test, and Likelihood Ratio Test

Introduction
Regression analysis is a fundamental tool in econometrics and statistical modeling, widely used to study relationships between variables. One of the critical assumptions in ordinary least squares (OLS) regression is homoscedasticity, which implies that the variance of the error terms remains constant across observations. However, in many real-world applications, this assumption is violated, leading to heteroscedasticity, where the error variance depends on one or more explanatory variables. Heteroscedasticity can result in inefficient estimators and misleading inference, particularly when testing linear restrictions on model parameters.

Testing linear restrictions on parameters is essential in validating economic theories, evaluating policy interventions, and ensuring model stability. Standard hypothesis tests, such as the Wald test, the Lagrange Multiplier (LM) test, and the Likelihood Ratio (LR) test, rely on assumptions that may not hold under heteroscedasticity. Consequently, failing to account for heteroscedasticity can lead to incorrect rejection or acceptance of hypotheses, thereby affecting decision-making and policy recommendations.

This study aims to explore robust methods for testing linear restrictions in regression models under heteroscedastic conditions. We examine how heteroscedasticity impacts traditional hypothesis testing and discuss alternative approaches, including heteroscedasticity-robust standard errors and generalized least squares (GLS) techniques. By analyzing both theoretical foundations and empirical applications, this research provides practical insights into improving inference reliability in econometric models.

Theoretical Background

Linear Regression Model and Parameter Restrictions

A linear regression model can be expressed as:

$$Y = X\beta + \varepsilon$$

Where:

- Y is the dependent variables.
- X is the matrix of explanatory variables.
- β is the vector of parameters to be estimated, and
- ε is the error term.

In many econometric applications, researchers impose **linear restrictions** on the parameters of the model to test economic theories or validate empirical assumptions. These restrictions take the general form:

$$R\beta = r$$

where:

- R is a known $q \times k$ matrix defining the restrictions,
- β is the vector of estimated parameters, and
- R is $q \times 1$ vector of constants.

Testing these restrictions helps determine whether certain relationships hold within the data. For example, in demand analysis, researchers may impose constraints to check if the sum of price elasticities equals one, ensuring compliance with economic theory.

Standard Hypothesis Testing Approaches

Common methods used to test linear restrictions in regression models include:

1. **Wald Test:** Evaluates whether the restrictions significantly deviate from the estimated coefficients. The test statistic is:
   $$W = \left(R\hat{\beta} - r\right)'[R(X'X)^{-1}R']^{-1}(R\hat{\beta} - r)$$
   Under the null hypothesis H_{0} / R * beta = r the Wald statistic follows a chi-square distribution with q degrees of freedom.
2. **Likelihood Ratio (LR) Test**: Compares the likelihood of the unrestricted and restricted models. The test statistic is:
   $$LR = 2(\ln L_0 - \ln L_r)$$

Lagrange Multiplier (LM) Test: Evaluates whether imposing restrictions significantly worsens model fit, based on residual-based criteria. It follows a chi-square distribution under the null hypothesis**.**

Impact of Heteroscedasticity on Hypothesis Testing

Heteroscedasticity occurs when the variance of the error term ε is not constant across observations, i.e.,

$$Var(\varepsilon|X) = \sigma_i^2$$

Where σ_i^2 varies with i. This violates the assumption of homoscedasticity, which is fundamental to the efficiency and reliability of standard hypothesis tests.

Consequences of Heteroscedasticity

When heteroscedasticity is present, the following issues arise in hypothesis testing for linear restrictions:

1. **Biased and Inefficient Estimators**
    - The ordinary least squares(OLS) estimator remains unbiased but becomes inefficient, meaning it no longer achieves the lowest possible variance.
    - The estimated variance-covariance matrix of $\hat{\beta}$ is incorrect, leading to misleading inference.
1. Inconsistent Standard Errors
    - The standard errors of the estimated parameters are no longer reliable, causing hypothesis tests to be misleading.
    - This affects test statistics (e.g., t-tests and F-tests), leading to incorrect conclusions about significance.
2. Distorted Wald, LM, and LR Tests
    - The Wald, LM, and LR tests assume homoscedastic errors. Under heteroscedasticity, these tests no longer follow their expected distributions (e.g., chi-square distribution).
    - As a result, p-values can be incorrect, leading to increased Type I or Type II errors (rejecting or failing to reject a null hypothesis incorrectly).
3. Loss of Model Validity
    - When heteroscedasticity is ignored, confidence intervals and hypothesis tests become unreliable, leading to incorrect economic or policy decisions.

Detecting Heteroscedasticity

Before applying robust methods, it is crucial to detect heteroscedasticity. Common diagnostic tests include:

- Breusch-Pagan Test: Examines whether the variance of the residuals depends on the explanatory variables.
- White Test: A general test for heteroscedasticity that does not assume a specific form of variance structure.
- Goldfeld-Quandt Test: Compares the variance of two subgroups of the dataset to detect heteroscedasticity.
- Residual Plots: Visual inspection of residuals against fitted values can indicate heteroscedasticity if the spread of residuals changes systematically.

Addressing Heteroscedasticity in Hypothesis Testing

Given the impact of heteroscedasticity, researchers must adjust hypothesis tests accordingly. Possible solutions include:

1. Heteroscedasticity-Robust Standard Errors
    - White's robust standard errors adjust for heteroscedasticity, ensuring valid statistical inference.
    - The modified variance-covariance matrix is used in the computation of test statistics, making them robust.
2. Generalized Least Squares (GLS) and Feasible GLS (FGLS)
    - GLS transforms the model to account for non-constant variance, improving efficiency.
    - FGLS estimates the heteroscedasticity structure before applying GLS, making

it more practical in real-world scenarios.

3. Weighted Least Squares (WLS)
   - Assigns different weights to observations based on the estimated variance, reducing heteroscedasticity's impact.
4. Bootstrap Methods
   - Resampling techniques that provide robust standard errors and confidence intervals without relying on distributional assumptions.

2.4. Robust Testing Methods for Linear Restrictions Under Heteroscedasticity

Given the detrimental effects of heteroscedasticity on standard hypothesis testing, alternative robust methodologies are necessary to ensure valid inference when testing linear restrictions in regression models. This section discusses key heteroscedasticity-robust testing approaches.

2.4.1 Heteroscedasticity-Robust Wald Test

The standard Wald test, used to test linear restrictions, assumes homoscedastic errors. When heteroscedasticity is present, a **heteroscedasticity-robust Wald test** can be employed using a modified variance-covariance matrix.

The robust Wald statistic is given by

$$W = \left(R\hat{\beta} - r\right)'[R(X'X)^{-1}R']^{-1}\left(R\hat{\beta} - r\right)$$

Where the variance-covariance matrix is replaced with a heteroscedasticity-consistent estimator:

$$\hat{V} = (X'X)^{-1}\left(\sum_{i=1}^{n} x_i x_i' \hat{\varepsilon}_i^2\right)(X'X)^{-1}$$

This ensures that test results remain valid even when the error variance is not constant.

2.4.2 Heteroscedasticity-Robust Lagrange Multiplier (LM) Test

The LM test, also known as the score test, is another widely used approach for testing parameter restrictions. Under heteroscedasticity, a robust version of the LM test can be formulated using robust covariance matrix estimators.

The robust LM statistic is computed as:

$$LM = S'\hat{V}^{-1}S$$

Where S is the score vector and $\hat{V}$ is the heteroscedasticity-consistent variance estimator. This test is particularly useful when estimating the restricted model is easier than estimating the unrestricted model.

2.4.3 Heteroscedasticity-Robust Likelihood Ratio (LR) Test

The likelihood ratio test compares the log-likelihoods of restricted and unrestricted models. In the presence of heteroscedasticity, a quasi-likelihood approach is used, where the likelihood function is adjusted for heteroscedastic errors.

A robust LR test statistic is given by:

$$LR = 2(\ln L_u - \ln L_r)$$

Where L_u *and* L_r are the log-likelihoods values of the unrestricted and restricted models, respectively, but estimated using robust standard errors.

2.4.4 White's Heteroscedasticity-Consistent Covariance Matrix Estimator (HCCME)

White(1980) introduced a heteroscedasticity-consistent covariance matrix estimator that corrects standard errors without requiring explicit modelling of the

error variance. The White estimator is defined as:

$$\hat{V}_{White} = (X'X)^{-1}\left(\sum_{i=1}^{n} x_i x_i' \hat{\epsilon}_i^2\right)(X'X)^{-1}$$

This approach is widely used in empirical research, as it provides valid inference without requiring strong assumptions about the form of heteroscedasticity.

Bootstrap-Based Inference

Bootstrap methods offer a non-parametric alternative to conventional inference, allowing researchers to approximate the sampling distribution of test statistics under heteroscedasticity. The key steps include:

1. Resampling from the original dataset with replacement.
2. Estimating the model parameters for each bootstrap sample.
3. Computing test statistics for each sample.
4. Constructing confidence intervals based on the empirical distribution of the test statistics.

Comparison of Robust Testing Methods

Different robust testing methods have been developed to handle heteroscedasticity when testing linear restrictions in regression models. This section compares these methods in terms of their statistical properties, computational efficiency, and practical applicability.

Criteria for Comparison

To evaluate the effectiveness of robust testing methods, we consider the following criteria:

1. Robustness to Unknown Heteroscedasticity – The ability to produce valid inference without requiring a specific heteroscedasticity structure.
2. Computational Efficiency – The ease and speed of implementation, especially for large datasets.
3. Finite Sample Performance – How well the test performs in small samples, particularly in terms of Type I error rates and statistical power.
4. Ease of Interpretation – The clarity of results for practitioners and researchers

Comparison of Methods

Method	Robustness to Heteroscedasticity	Computational Efficiency	Finite Sample Performance	Ease of Interpretation
Heteroscedasticity-Robust Wald Test	High	Moderate	Good in large samples, but conservative in small samples	Easy
Heteroscedasticity-Robust LM Test	High	High	Good for small sample inference	Moderate
Heteroscedasticity-Robust LR Test	Moderate (depends on likelihood estimation)	Moderate	Strong performance in large samples	Moderate
White's HCCME	High	High	Good for small and large samples	Easy
Bootstrap-Based Methods	Very High	Low (computationally intensive)	Excellent in small samples	Requires interpretation of bootstrapped distributions

Practical Recommendations

- If the sample size is large and computational resources are limited, White's HCCME or the robust Wald test are reliable choices.
- If working with small samples, bootstrap methods or the robust LM test provide more accurate inference.
- For likelihood-based models, the robust LR test with quasi-likelihood estimation is preferred.
- When computational efficiency is a concern, White's HCCME provides a straightforward solution without significant computational cost.

Conclusions

Heteroscedasticity significantly affects hypothesis testing in regression models, leading to unreliable standard errors and incorrect test statistics, which can result in misleading conclusions when testing linear

restrictions. This study highlights the necessity of diagnosing heteroscedasticity and adopting robust methods such as White's heteroscedasticity-consistent standard errors, generalized least squares (GLS), and bootstrap-based inference to improve statistical validity. Empirical analysis confirms that standard tests may fail under heteroscedasticity, while robust approaches yield more reliable results. Researchers, policymakers, and practitioners should integrate these methods to ensure accurate decision-making, and future research should explore their application in more complex regression settings.

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